Nuclear Reaction Theory

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Theoretical predictions of residue cross-sections for the superheavy elements

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1. Introduction

Productions of the superheavy elements are predicted with the two-step model for fusion of massive systems [1], combined with the theory of the statistical decay of the compound nucleus. As is well known, the fusion of lighter heavy-ion systems is determined by whether the incident system can enter the inner side of the Coulomb barrier or not, but in heavy systems an additional process is indispensable for the formation of the compound nucleus, because there is a conditional saddle point which locates between the position of the Coulomb barrier top and the spherical shape of the compound system. Therefore, the fusion of massive heavy-ion systems which are necessary for the synthesis of superheavy elements (SHE) requires two steps as schematically shown in the first figure; firstly the approaching phase up to the contact of the incident ions after overcoming the Coulomb barrier and secondly the shape evolution phase to the spherical compound nucleus, starting from the pear-shaped composite system made by the projectile and the target of the incident channel. Thus, the product of the sticking and the formation probabilities (P_{stick} and P_{form}), which are obtained by solving dynamics in the two steps, respectively, gives the fusion probability P_{fusion}.

$$P^{J}_{fusion}(E_{c.m.}) = P^{J}_{stick}(E_{c.m.}) \times P^{J}_{form}(E_{c.m.}) \quad (1)$$

where J denotes a total spin of the system and $E_{c.m.}$ a c.m. incident energy. The amalgamated system is expected to be excited internally, that is, the incident kinetic energy is transferred to a thermal energy. The dissipation of the incident kinetic energy may start before the top of the Coulomb barrier, or may start at the moment of touching of the two nuclear matters. If we presume the latter case, the sticking probability is given by a quantum-mechanical barrier penetration factor or simply by a step function at the barrier height, and the formation probability should be calculated by a Langevin equation with the initial momentum being defined by the incident kinetic energy and with a time-dependent temperature which describes a heating-up process. On the other hand, if we presume the former case, the sticking probability should be calculated by a Langevin equation with a frictional force and with a time-dependent temperature, and the formation probability is calculated with a Langevin equation with a constant temperature for shape evolution, if the incident energy is mostly damped at the moment of the contact or of the amalgamation. In view of the results of the Deep-Inelastic Collisions (DIC), it is natural to presume the former. Of course, one would be interested in the enhancement in the sub-barrier energy which is known to originate from coherent couplings with some other channels and to be very important in lighter heavy-ion systems. But, taking into account a strong fusion hindrance which is well known to exist in massive systems, an incoherent treatment, i.e., a treatment of effects of the couplings as a friction would be reasonable.

2. Approaching Phase

We employ a classical treatment for the description of the relative motion of the incident ion system with a frictional force and an associated fluctuation force which is missing in the original classical treatments [2, 3]. The equation is as follows,

$$
\begin{cases}
m\dfrac{d^2r}{dt^2}=-\dfrac{\partial V}{\partial r}-\dfrac{\partial}{\partial r}\dfrac{\hbar^2L^2}{2\mu r^2}-C_r(r)\dfrac{dr}{dt}+R_r(t)\\
\dfrac{dL}{dt}=-\dfrac{C_T(r)}{\mu}[L-\dfrac{5}{7}L_0]+R_T(t)
\end{cases}
\qquad (2)
$$

$$\langle R_i(t)R_j(t)\rangle=2T(t)\cdot C_i(r(t))\cdot\delta_{ij}\cdot\delta(t-t')$$

where m is the reduced masse and V is the sum of the Coulomb and the nuclear attrative potentials. $C_i(r)$ is the radial and tangenatial frictions, respectively, where the rolling friction is neglected.L_0 is the incident angular momentum and 5/7 L_0 so called sliding limit. R(t) denotes a Gaussian random force with zero mean value. The last equation is the dissipation-fluctuation theorem assumed and in case i=j=r, r^2 factor is necessary for $C_i(r)$. As for the friction in the approaching phase, there are two models available; one is the surface friction model (SFM) [2] and the other the proximity friction [4]. The former is rather successful in reproducing DIC data except spins of outgoing fragments which appears to require the inclusion of the rolling friction. And the strengths are very different between the radial and tangential frictional forces, which may be unnatural. On the other hand, the latter does not take into account effects of strong couplings to inelastic channels which are important in low energy. Thus, no outstandingly good model for the friction in the approaching phase is available for the moment. Thus, these models were used to calculate sticking probabilities [5]. It turned out that SFM is much stronger than the proximity one, i.e., sticking probabilities calculated by SFM are extremely small, though which one is realistic is not determined yet. At the same time, there is a common feature that the radial momentum has a distribution with a Gaussian shape which would be due to the assumption of the Gaussian random forces associated with the frictional force. The width of the distribution is consistent with the temperature determined by the internal energy transferred from the kinetic energy. The distribution is used for the initial condition for the dynamics of the second phase, i.e., for shape evolution toward the spherical shape. In this sense the model should be called a stochastic two-step model.

3. Shape Evolution

As stated in the Introduction, the amalgamated system locates at the outside of the conditional saddle point or at the outside of the ridgeline. Therefore, in order to obtain the formation probability, we have to solve shape evolution towards the spherical shape under a frictional force and its associated random force. The ratio between the number of trajectories that pass over the ridge line and the total number gives the formation probability, though most of them return back to reseparation due to the conservative potential calculated with the liquid drop model (LDM). An example of LDM energy surface is shown in the second figure for the system with A=272 and Z=112. Shapes are specified in terms of the two-center parameterization (R/R_0,a) with the neck parameter fixed to be 1.0 [6]. The cross in the figure denotes the contact configuration of $^{64}Ni+^{208}Pb$, i.e., the starting point of the evolution.

The dynamics is again described by a Langevin equation, generally multi-dimensional one,

$$\begin{cases} \frac{dq_i}{dt} = (m^{-1})_{ij} \cdot p_j \\ \frac{dp_i}{dt} = -\frac{\partial V^J}{\partial q_i} - \frac{1}{2}\frac{\partial}{\partial q_i}(m^{-1})_{jk} \cdot p_j p_k - \gamma_{ij} \cdot (m^{-1})_{jk} \cdot p_k + g_{ij} \cdot R_j(t) \end{cases}$$

$$\langle g_{ik} \cdot g_{jk} \rangle = \gamma_{ij} \cdot T^J \quad (3)$$

where V^Jdenotes the LDM potential plus the centrifugal force for spin J. The last equation is again the dissipation-fluctuation theorem with a constant temperature T^J for spin J. We calculate the friction tensor by employing so-called one-body model (OBM), i.e., one-body wall-and-window formula [7] with the two-center parameterization of nuclear shapes. For a given initial momentum conjugate with the distance, we calculate many trajectories, some of which pass over the ridgeline. Then, the formation probability is given by the average over the initial momentum distribution obtained in the first step.

4. Residue Cross Sections

Assuming the compound nucleus theory of reactions, residue cross sections of the superheavy elements are given by a product of the fusion probability P_{fusion} and the survival probability P_{surv}as follows,

$$\sigma_{res} = \pi \lambda\!\!\!^{-2} \sum (2J+1) P^J_{fusion}(E_{c.m.}) \cdot P^J_{surv}(E^*) \quad (4)$$

where $E^*=E_{c.m.}+Q$ with the fusion Q-value. P_{surv}denotes the probability for the compound nucleus to survive against fission and charged particle emission. It is calculated by the new computer program which is constructed, based on the theory of the time-dependent statistical decay [8]. An essential parameter is the shell correction energies for the superheavy elements. Although there are many predictions on them by the structure calculations with various levels of nuclear model [9], but unfortunately they differ with each other. We introduce a single reduction factor 0.4 for P. Moeller et al's predictions, because their predictions mostly appear to be largest compared with others' in the absolute values for nucleides around Z=114. An example of the residue cross section is shown in the last figure for ^{64}Ni+^{208}Pb system leading to Z=112 element after one neutron emission. The result is compared with the available data [10]. Theory reproduces the data remarkably well. Similar results are obtained for ^{70}Zn+^{208}Pb, though the data to be compared are few. Predictions are made on Z=113 and 114 elements.

It should be worth to remind that the model has been already applied to the hot fusion path, i.e., ^{48}Ca+ actinide systems with the same parameters and resulted in a reasonabely good reproduction of the available data [1, 11].

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The physics of nucleus-nucleus fusion

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Twelve measured cross sections for one-neutron-out reactions with ^{208}Pb and ^{209}Bi targets and projectiles ranging from ^{48}Ca to ^{70}Zn are compared with a theoretical model. The model assumes that the cross section is the product of three factors: a) the cross section for the nuclei to stick, b) the probability for the system to diffuse ("up hill") over the barrier separating it from the compound-nucleus configuration and c) the probability for the compound nucleus to survive fission and the emission of a second neutron. With one parameter adjusted to have the value 1.6 fm (equal to the separation between the nuclear surfaces at which the diffusion process begins) the cross sections, ranging over 6 orders of magnitude, are reproduced adequately. The centroids and widths of the excitation functions are in good agreement with measurements. The model is used to calculate cross sections for even heavier reactions, using the same targets and ^{76}Ge, ^{82}Se and ^{86}Kr as projectiles.

Production of spherical super-heavy nuclei

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Introduction

Since GSI started its research program on super-heavy elements in the 1970s, an extensive program has continuously been carried out to explore the physics involved in the different processes which are relevant for the synthesis of the heaviest nuclei. Special emphasis was put on those features which are particularly relevant for reaching the next doubly magic spherical shell closure beyond ^{208}Pb. This long-term research program included several aspects, reaching from the amalgamation process of projectile and target nuclei under the influence of the strong repulsive Coulomb force up to the cooling down of the compound nucleus by particle evaporation in competition with a strong fission branch. It was decided to perform the necessary elaborate investigations by studying the synthesis of proton-rich nuclei near the 126-neutron shell, whose formation can be considered as a realistic test case for the production of spherical super-heavy nuclei. The common features are the spherical shape of the nuclear ground state and a large shell effect of more than 5 MeV. The larger liquid-drop component of the fission barrier of a few MeV ensures sizeable formation cross sections, which are a pre-requisite for systematic studies.

Entrance channel

Specific features of the entrance channel arise in the synthesis of spherical super-heavy nuclei, because the projectile-target combinations available on the basis of primordial nuclei require rather mass-symmetric systems, which consequently experience a very strong Coulomb repulsion in the amalgamation phase. The most extensive exploration of entrance-channel effects in fusion of massive systems, extending from ^{90}Zr + ^{90}Zr to ^{110}Mo + ^{110}Mo, is documented in Ref. [1]. These experiments revealed the onset of a considerable reduction of the fusion probability near the potential barrier already for symmetric systems with $Z_1 * Z_2 = 1600$. This hindrance was related to the "extra-push" phenomenon, postulated by W. Swiatecki. More massive systems showed a stronger fusion hindrance and a more gradual increase of the fusion probability as a function of bombarding energy. The gradual increase of the fusion probability was interpreted as an evidence for strong fluctuations connected with the extra push. The nuclear-structure properties of projectile and target were found to have a decisive influence on this phenomenon: Projectile-target combinations with nuclei close to major shells were less affected. Some new experiments performed in other laboratories shed new light on these results. H. Ikezoe *et al*. have shown that the fusion probability of a deformed projectile or target nuclei can be understood by variations in the orientation of the colliding nuclei [2]. Tip-on-tip configurations lead to less compact contact configurations and experience a stronger extra-push in contrast to side-on-side collisions. In another very important work, Hinde *et al*. completed the series of projectile-target combinations leading to the compound nucleus ^{220}Th studied in [1] by the system ^{16}O + ^{204}Pb [3]. They obtained the surprising result that already the system ^{40}Ar + ^{174}Hf, which was considered previously to fuse in the lower angular-momentum range with high probability, experiences a strong fusion hindrance. On the basis of this new result, the systematics of fusion probabilities deduced in ref. [1] is to be revised. As a general conclusion, the extra-push phenomenon is found to set in for considerably lighter systems than previously known.

Exit channel

The exit channel in the synthesis of spherical super-heavy nuclei is subject to very specific features, which may differ considerably from those met in the production of deformed super-heavy nuclei. In the de-excitation of slightly excited highly fissile nuclei, nuclear-structure phenomena are expected to govern the competition between particle evaporation and fission due to the influence of shell effects and collective properties on the level density. Therefore, any extrapolation based on the experience in the synthesis of the heaviest nuclei reached up to now is highly doubtful. Again, extended studies on the synthesis of proton-rich nuclei near the 126-neutron shell were chosen as the appropriate tools to explore the features of the exit channel in the production of spherical super-heavy nuclei.

The results of a first experimental program, based on the production of a series of compound nuclei around the 126-neutron shell by heavy-ion fusion reactions, are documented in ref. [1]. It was found that the strong ground-state shell effect of these nuclei does not enhance the survival probability of the fused system against fission, although it is responsible for about half the height of the fission barrier.

In order to avoid the influence of fusion hindrance on these results, the fission competition of these nuclei was studied recently with a different experimental approach. The nuclei of interest were produced as secondary beams. They were excited by electromagnetic interactions to states of low angular momentum at energies only slightly above the fission barrier, and the cross sections for consecutive fission were measured [4]. Even at these low excitation energies no influence of the 126-neutron shell on the fission probability was observed.

A third approach exploited the production of proton-rich nuclei near the 126-neutron shell by bombarding copper, hydrogen and deuterium nuclei with ^{238}U at 1 A GeV [5-7]. In these reactions, a field of nuclides slightly lighter than the projectile are produced with excitation energies extending to several hundred MeV or more. The production of these prefragments is certainly not influenced by nuclear-structure properties. Only in the last steps of the deexcitation process an eventual influence of the 126-neutron shell on the fission competition is expected, which would lead to a structure in the nuclide distribution observed. Since the spallation process produces a large field of nuclei with comparable cross sections, it is well suited to reveal a possible enhancement of the survival probability of nuclei near N=126 in the deexcitation process by the appearance of a ridge which would be superimposed on the broad distribution of nuclide cross sections formed by the spallation process. The results of this approach also agreed with those found in our preceding experiments: The large ground-state shell effect of N=126 nuclei does not lead to a noticeable enhancement of the survival probability against fission in the deexcitation process.

These findings have been traced back to the specific features of the level densities of magic spherical nuclei [4,5]. While the number of intrinsic excitations is influenced by the sequence of single-particle energies, which would strongly enhance particle emission with respect to fission, the spherical nuclear shape only allows for collective excitations of vibrational character. Compared to the large number of rotational levels found at the fission-barrier deformation, this leads to strong enhancement of fission. It seems that these two counteracting structural effects cancel to a great extent in the case of proton-rich N=126 nuclei. As a consequence for the production of spherical super-heavy nuclei, we expect qualitatively a similar effect. Therefore, we expect that the systematics of production cross sections found in the synthesis of deformed super-heavy nuclei cannot be extrapolated for estimating the production cross sections of spherical super-heavy nuclei. Instead, one will probably face a considerably stronger decrease in the formation cross sections in the transition from the deformed to the spherical super-heavy region.

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Fusion-fission dynamics and perspectives of superheavy element formation

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The interest in the synthesis of super-heavy nuclei has lately grown due to the new experimental results [1] demonstrating a real possibility of producing and investigating the nuclei in the region of the so-called "island of stability". The new reality demands a more substantial theoretical support of these expensive experiments, which will allow a more reasonable choice of fusing nuclei and collision energies as well as a better estimation of the cross sections and unambiguous identification of evaporation residues (ER). The talk will focus on reaction dynamics of superheavy nucleus formation and decay at beam energies near the Coulomb barrier. The aim will be to review the things we have learned from recent experiments [1,2] on fusion-fission reactions leading to the formation of compound nuclei with Z^3102 and from their extensive theoretical analysis [3-6]. Major attention is paid to the dynamics of formation of very heavy compound nuclei taking place in strong competition with the process of fast fission (quasi-fission). The choice of collective degrees of freedom playing a principal role, finding the multi-dimensional driving potential and the corresponding dynamic equations of motion regulating the whole process are discussed along with a new approach proposed in [3,5] to description of fusion-fission dynamics of heavy nuclear systems based on using the two-center shell model idea. A possibility of deriving the fission barriers of superheavy nuclei directly from performed experiments is of particular interest here. In conclusion the results of detailed theoretical analysis of available experimental data on the "cold" and "hot" fusion-fission reactions will be presented. Perspectives of future experiments will be discussed along with additional theoretical studies in this field needed for deeper understanding of the fusion-fission processes of very heavy nuclear systems.

A whole process of super-heavy nucleus formation can be divided into three reaction stages. At first stage colliding nuclei overcome the Coulomb barrier and approach the point of contact $R_{cont}=R_1+R_2$. Quasi-elastic and deep-inelastic reaction channels dominate at this stage leading to formation of projectile-like and target-like fragments (PLF and TLF) in exit channel. At sub-barrier energies only small part of incoming flux with low partial waves reaches the point of contact. Denote the corresponding probability as $P_{cont}(l,E)$. At the second reaction stage touching nuclei evolve into the configuration of almost spherical compound mono-nucleus. For light or very asymmetric nuclear systems this evolution occurs with a probability close to unity. Two touching heavy nuclei after dynamic deformation and exchange by several nucleons may re-separate into PLF and TLF or may go directly to fission channels without formation of compound nucleus. The later process is usually called quasi-fission. Denote a probability for two touching nuclei to form the compound nucleus as $P_{CN}(l,E)$. At third reaction stage the compound nucleus emits neutrons and γ-rays lowering its excitation energy and forming finally the residual nucleus in its ground state. This process takes place in strong competition with fission (normal fission), and the corresponding survival probability $P_{xn}(l,E^*)$ is usually much less than unity even for low-excited superheavy nucleus.

Thus, the production cross section of a cold residual nucleus B, which is the product of neutron evaporation and γ-emission from an excited compound nucleus C, formed in the fusion process of two heavy nuclei $A_1+A_2\to C\to B+xn+N\gamma$ at center-of mass energy E close to the Coulomb barrier in the entrance channel, can be decomposed over partial waves and written as

$$\sigma^{xn}_{ER}(E) \approx \frac{\pi\hbar^2}{2\mu E}\sum_{l=0}^{\infty}(2l+1)P_{cont}(l,E)P_{CN}(A_1+A_2\to C;l,E)P_{xn}(C\to B;l,E^*) \qquad (1)$$

Different theoretical approaches are used for analyzing all the three reaction stages. However, the dynamics of the intermediate stage of the compound nucleus formation is the most vague. Setting here $P_{xn}=1$ we get the cross section of CN formation σ_{CN}, which can be measured by detection of ERs and fission fragments forming in normal fission (if they are distinguished from quasi-fission fragments and from products of deep inelastic collision). Setting in addition $P_{CN}=1$ we get the capture cross section σ_{cap}, which can be measured by detection of all fission fragments (if they are distinguished from products of deep inelastic collision). For symmetric fusion reactions σ_{CN} and σ_{cap} cannot be measured experimentally.

Coupling with the excitation of nuclear collective states (surface vibrations and/or rotation of deformed nuclei) and with nucleon transfer channels significantly influences the capture cross section at near-barrier energies. Incoming flux has to overcome in fact the multi-dimensional ridge with the height depending on orientation and/or dynamic deformation. In [3,4] a semi-empirical approach was proposed for calculating the penetration probability of such multi-dimensional potential barriers. The capture cross sections calculated within this approach are shown in Fig. 1 for the three reactions (solid curves). They are compared with theoretical calculations made within a model of one-dimensional barrier penetrability for spherical nuclei (dashed curves). In all three cases a substantial increase in the barrier penetrability is observed in the sub-barrier energy region. Good agreement between the calculated and experimental capture cross sections allows us to believe that we may get rather reliable estimation of the capture cross section for a given projectile-target combination if there are no experimental data or these data cannot be obtained at all (symmetric combinations).

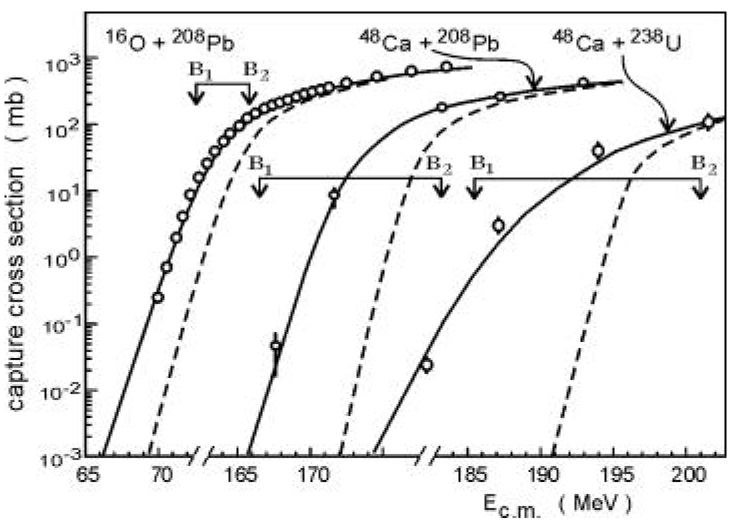

Figure 1. Capture cross sections in the $^{16}O+^{208}Pb$, $^{48}Ca+^{208}Pb$, and $^{48}Ca+^{238}U$ fusion reactions. Dashed curves represent one-dimensional barrier penetration calculations with the Bass barriers. Solid curves show the effect of dynamic deformation of nuclear surfaces (two first reactions) and orientation of statically deformed nuclei ($^{48}Ca+^{238}U$ case).

The processes of the compound nucleus formation and quasi-fission are the least studied stages of heavy ion fusion reaction. To solve the problem we have to answer very principal questions. What are the main degrees of freedom playing most important role at this reaction stage? What is the corresponding driving potential and what are appropriate equations of motion for description of time evolution of nuclear system at this stage? Today there is no consensus for the answers and for the mechanism of the compound nucleus formation itself, and quite different, sometimes opposite in their physics sense, models are used for its description. In [3,5] a new approach was proposed for description of fusion-fission dynamics based on a simplified semi-empirical version of the two-center shell model idea [7]. It is assumed that on a way from the initial configuration of two touching nuclei to the compound nucleus configuration and on reverse way to the fission channels the nuclear system consists of two cores (Z_1,N_1) and (Z_2,N_2) surrounded with a certain number of common (shared) nucleons ΔA=ACN-A1-A2 moving in the whole volume occupied by the two cores. The processes of compound nucleus formation, fission and quasi-fission take place in the space (Z_1,N_1,β_1;Z_2,N_2,β_2), where β_1 and β_2 are the dynamic deformations of the cores. The compound nucleus is finally formed when two fragments A_1 and A_2 go in its volume, i.e., at $R(A_1)+R(A_2)=RCN$ or at $A_1^{1/3}+A_2^{1/3}=ACN^{1/3}$.

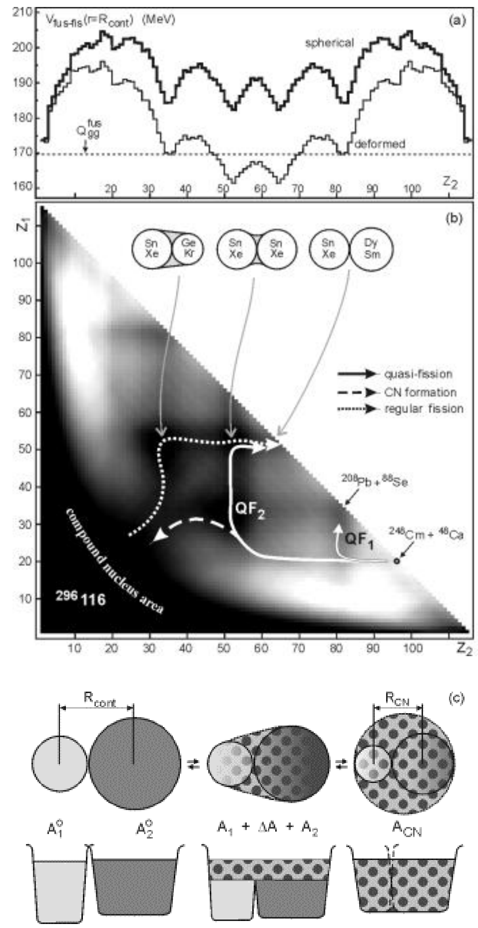

Figure 2. Driving potential $V_{fus\text{-}fis}(Z_1,Z_2)$ of the nuclear system consisting of 116 protons and 180 neutrons. (a) Potential energy of two touching nuclei at $A_1+A_2=A_{CN}$, ΔA=0, i.e., along the diagonal of the lower figure. The thick line corresponds to the case of spherical nuclei, whereas the thin line corresponds to $\beta_1+\beta_2$=0.3. (b) Topographical landscape of the driving potential on the plane (Z_1, Z_2) (zero deformations). The dark regions correspond to the lower potential energies (more compact configurations). (c) Schematic view of the process of compound nucleus formation, fission and quasi-fission in the space of A_1, A_2 and ΔA, i.e., the number of nucleons in the projectile-like nucleus, target-like nucleus, and shared nucleons, here $A_1+A_2+\Delta A=A_{CN}$.

The corresponding driving potential $V_{fus\text{-}fis}(r, Z_1, N_1, \beta_1, Z_2, N_2, \beta_2)$ was derived in [3] and is shown in Fig. 2 as a function of Z_1, Z_2 (minimized over N_1, N_2 and at fixed values of $\beta_1+\beta_2$). There are several advantages of the proposed approach. The driving potential is derived basing on experimental binding energies of two cores, which means that the "true" shell structure is taken into account. The driving potential is defined in the whole region $R_{CN} < r < \infty$, it is a continuous function of r at $r=R_{cont}$, and it gives the realistic Coulomb barrier at $r=R_B>R_{cont}$. At last, instead of using the variables (A_1,A_2), we may easily recalculate the driving potential as a function of mass asymmetry $(A_1-A_2)/(A_1+A_2)$ and elongation $R_{12} = r_0\,(A_1^{1/3}+A_2^{1/3})$ (at $r > R_{cont}$, $R_{12} = r = s+R_1+R_2$, where s is the distance between nuclear surfaces). These variables along with deformation $\beta_1+\beta_2$ are commonly used for description of fission process. The corresponding driving potential is shown in Fig. 3.

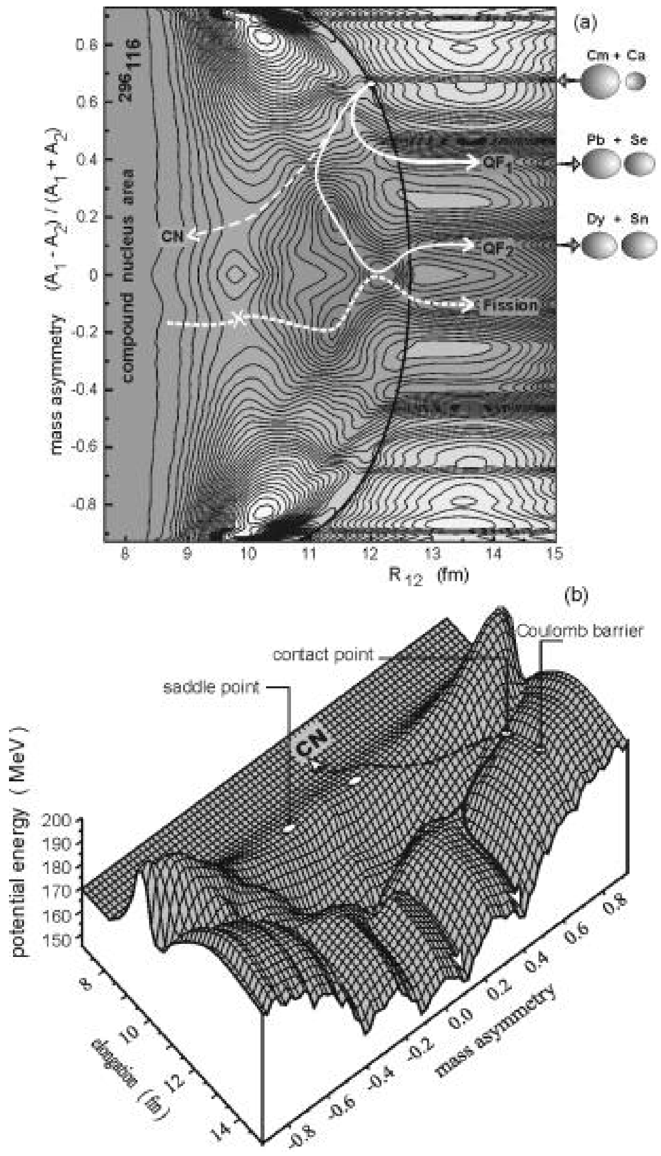

Figure 3. Driving potential $V_{fus\text{-}fis}$ as a function of mass asymmetry and distance between centers of two nuclei with the deformations $\beta_1+\beta_2=0.3$, topographical landscape (a) and three-dimensional plot (b). The black solid curve in (a) shows the contact configurations. The paths QF_1 and QF_2 lead to the asymmetric and near-symmetric quasi-fission channels, the dashed curve shows the most probable way to formation of the compound nucleus, and dotted curve corresponds to the normal (regular) fission. See the conformity with Fig. 2.

As can be seen from Fig. 2 and Fig. 3, the shell structure, clearly revealing itself in the contact of two nuclei (Fig. 2a), is also retained at $\Delta A \neq 0$ ($R_{12}<R_{cont}$), see the deep minima in the regions of $Z_{1,2}$—50 and $Z_{1,2}$—82 in Fig. 2b. Following the fission path (dotted curves in Fig. 2b and Fig. 3) the system overcomes a multi-humped fission barrier, which is well known in fission dynamics. The intermediate minima correspond to the shape isomeric states. From our analysis we may definitely conclude that these isomeric states are nothing else but two-cluster configurations with magic or semi-magic cores (see the inset in Fig. 2b).

As regards the superheavy compound nucleus formation in the fusion reaction ^{48}Ca+^{248}Cm, one can see that after the contact, the nuclear system may easily decay into the quasi-fission channels (mainly asymmetric: Se+Pb, Kr+Hg and also near-symmetric: Sn+Dy, Te+Gd) - solid arrow lines in Fig. 2b and Fig. 3. Only a small part of the incoming flux reaches a compound nucleus configuration (dashed arrow line). The experimental data on the yield of quasi-fission fragments in collisions of heavy nuclei [2] were found quite understandable in terms of multi-dimensional potential energy surface shown in Fig. 2 and Fig. 3 [6].

Using the driving potential $V_{fus\text{-}fis}(Z_1, N_1, \beta_1, Z_2, N_2, \beta_2)$ we may determine the probability of the compound nucleus formation $P_{CN}(A_1+A_2 \to C)$, being part of expression (1) for the cross section of the synthesis of super-heavy nuclei. It was by solving the transport equation for the distribution function $F(Z_1, N_1, \beta_1, Z_2, N_2, \beta_2;t)$. The probability of the compound nucleus formation is determined as an integral of the distribution function over the region $R_1+R_2 <= R_{CN}$. Similarly one can define the probabilities of finding the system in different channels of quasi-fission, i.e., the charge and mass distribution of fission fragments measured experimentally. Results of such calculations demonstrate quite reasonable agreement with the corresponding experimental data.

The detailed theoretical analysis of available experimental data on the "cold" and "hot" fusion-fission reactions has been performed and the cross sections of superheavy element formation have been calculated up to Z_{CN}=120 as well as the mass and charge distributions of quasi-fission fragments obtained in these reactions. The corresponding excitation functions for 2n, 3n, and 4n evaporation channels were calculated depending on different theoretical estimations of the neutron separation energies and fission barriers of superheavy nuclei. Optimal beam energies were found for production of the cold evaporation residues of new elements in the "hot" fusion reactions. We found also a possibility of deriving the fission barriers of superheavy nuclei directly from analysis of experimental data on the fusion-fission cross sections and from experimental data on the survival probability of those nuclei in evaporation channels of 3 and 4 neutron emission. In particular, the lower limits that we have obtained for the fission barrier heights of $^{283\text{-}286}$112, $^{288\text{-}292}$114 and $^{292\text{-}296}$116 nuclei are 5.5, 6.7 and 6.4 MeV respectively [6], which are really quite high resulting in relatively high stability of these nuclei.

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The cold formation and decay of superheavy nuclei including the orientation degree of freedom

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Theoretically, cold synthesis of new and superheavy elements (SHE) was proposed by one of us and collaborators [1-3] as back as in 1974-75, where a method was given for selecting out an *optimum* cold target-projectile combination. Cold compound systems were considered to be formed for *all* those target + projectile combinations that lie at the bottom of the *potential energy minima*, referred to as "cold reaction valleys" or reaction partners leading to "cold fusion" [2-6]. This information on cold reaction valleys was optimized [3] by the requirements of smallest interaction barrier, largest interaction radius and non-necked (no saddle) nuclear shapes, identifying the cases of "cold", "warm/ tepid" and "hot" fusion reactions. The theory, called the Quantum Mechanical Fragmentation Theory (QMFT), was advanced as a unified approach both for fission (including the cluster radioactivity (CR)) and heavy ion collisions. The key result behind the cold fusion reaction valleys (or the decay products in fission and CR) is the *shell closure effects* of one or both the reaction partners (or decay products). Also, fission and CR were considered as cold phenomenon on the basis of the QMFT, prior to their being observed experimentally as cold processes in 1980 and 1984, respectively [7].

In this paper, we review this theory via some new calculations based on the use of oriented and radioactive nuclei as beams and/ or targets. The use of neutron-rich radioactive nuclei is essential for overshooting the *center* of island of SHE (the next doubly magic nucleus) and the deformed oriented collisions could be useful since the fusion barrier gets lowered, or, in other words, the excitation energy of the cold compound system gets further reduced. As an example, we choose the recently used highly neutron-rich beam of ^{48}Ca on neutron-rich actinide targets ^{232}Th, ^{238}U, ^{242,244}Pu and ^{248}Cm, forming the compound systems $^{280}110^*$, $^{286}112^*$, $^{290,292}114^*$ and $^{296}116^*$ [8]. Note that the targets used are the deformed nuclei and, for near the Coulomb barrier energies, the compound nucleus excitation energy $E^* \sim 30$-35 MeV, in between the one for cold (10-20 MeV) and hot (40-50 MeV) fusion reactions. The resulting "warm/ tepid" compound systems de-excite by 3n and/ or 4n evaporations (and γ rays), compared to 1n and 2n in cold and 5n in hot fusion reactions, and give rise to new nuclei $^{277}110$, $^{283}112$, $^{287,288,289}114$ and $^{292}116$. These final nuclei are relatively long-lived and decay only via α-particles, giving the α-genetically related nuclei, called an α-decay chain. Then, as a second aim of this paper, we investigate the observed α–decay characteristics of these nuclei within the preformed cluster decay model (PCM) of Gupta and Collaborators [9-11] which is also based on the QMFT.

The QMFT is a dynamical theory of the three cold processes mentioned above, worked out in terms of the mass (charge) asymmetry $\eta=(A_1-A_2)/(A_1+A_2)$ ($\eta_Z=(Z_1-Z_2)/(Z_1+Z_2)$), the relative separation **R**, the deformations β_1^λ, β_2^λ (λ=2) of two nuclei or, in general, the fragments, and the neck parameter ε [12,13]. We extend the QMFT here to include the multipole deformation parameter λ=3 and 4 i.e. octupole and hexadecupole deformations also. In addition, we introduce two orientation angles θ_1 and θ_2 [14], see Fig. 1(a). So far, the time-dependent Schrödinger equation in η is solved for non-oriented collisions and for weakly coupled η and η_Z motions:

$$H\Psi(\eta,t)=i\hbar\frac{\partial}{\partial t}\Psi(\eta,t). \tag{1}$$

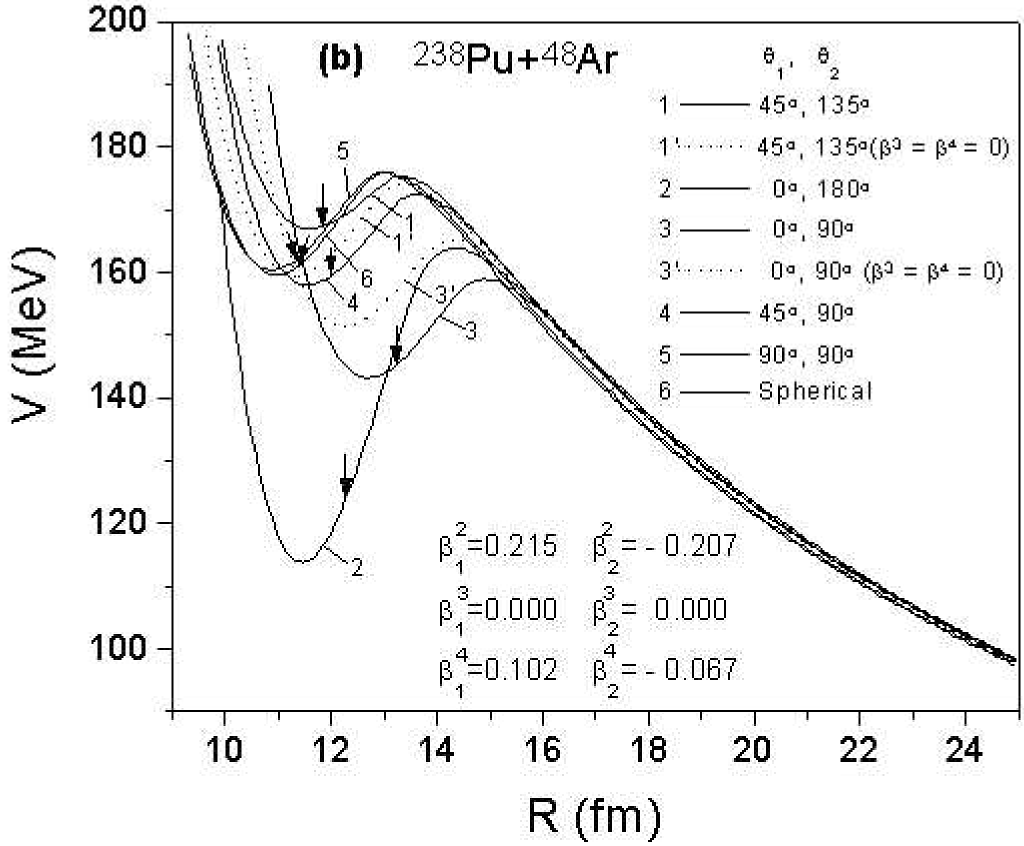

Figure 1. (a) Schematic configuration of two axially symmetric quadrupole deformed, oriented nuclei, in same plane. (b) Scattering potentials for ^{48}Ar+^{238}Pu at different orientations. Arrows denote R-values for s_0=1.0 fm.

Here R(t) is treated classically and the quadrupole deformations β_i^2 (i=1,2) and ε are fixed by minimizing the collective potential $V(R,\eta,\eta_Z, \beta_i^2,\varepsilon)$ in the above coordinates. Eq. (1) is solved for a number of heavy systems [13], which shows that for target + projectile combinations coming from *outside* the potential energy minima, a few nucleon to a large mass transfer occurs, whereas the same is zero for a target + projectile referring to potential energy minima. This means that *for cold reaction partners, the two nuclei stick together and form a deformed compound system*. A few nucleon transfer may, however, occur depending on whether a "conditional" saddle exists or not. Since the solution of Eq. (1) is very much computer-time consuming, the following simplifications are exercised based on calculated quantities.

The potentials $V(R,\eta)$ and $V(R,\eta_Z)$, calculated within the Strutinsky method i.e. $V=V_{LDM} + \delta U$, the liquid drop energy plus the shell effects calculated by using the asymmetric two-center shell model (ATCSM), show that the motions in both η and η_Z are much faster than the R-motion. This means that these both potentials are nearly independent of the R-coordinate and hence R can be taken as a time-independent parameter. This reduces the time-dependent Schrödinger equation (1) in η to the stationary Schrödinger equation in η,

$$\left\{-\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}}\frac{\partial}{\partial\eta}\frac{1}{\sqrt{B_{\eta\eta}}}+V_R(\eta)\right\}\Psi_R^{(\nu)}(\eta)=E_R^{(\nu)}\Psi_R^{(\nu)}(\eta) \quad (2)$$

where R is fixed at the post saddle point. This choice of R-value is justified by many good fits to both fission and heavy-ion collision data [7] and by an explicit, analytical solution of time-dependent Schrödinger equation in η_Z coordinate [15]. An interesting result of these calculations is that the yields ($\propto |\Psi(\eta)|^2$ or $|\Psi(\eta_Z)|^2$, respectively, for mass or charge distributions) are nearly insensitive to the detailed structure of the kinetic energy term in the Hamiltonian i.e. the Cranking masses $B_{\eta\eta}$ calculated consistently by using ATCSM. In other words, the static potential $V(\eta)$ or $V(\eta_Z)$ contain all the important information of a fissioning or colliding system. The positions of the minima are due to shell effects. Since these potentials are nearly independent of the choice of R-value, we have calculated them at some critical distance R_c where the two nuclei come in close contact with each other. We do the same here for oriented, neutron-rich radioactive nuclei having higher multipole deformations also.

For oriented nuclei, the potential $V(\eta)$ is the sum of binding energies (taken from Möller et al. [16] for $Z\geq 8$; and from experiments for $Z\leq 7$), the Coulomb and the proximity potential (both taken from [14] and extended to include higher multipole deformations) that depend on deformations as well as on orientations:

$$V(\eta)=\sum_{i=1}^{2}B\left(A_i,Z_i,\beta_i^{\lambda}\right)+E_c(Z_i,\beta_i^{\lambda},\theta_i)+V_P(A_i,\beta_i^{\lambda},\theta_i). \quad (3)$$

In Eq. (3) i=1,2 and λ =2,3,4. For the fixed orientations, the charges Z_1 and Z_2 in (3) are fixed by minimizing this potential in η_Z coordinate (which fixes the deformation coordinates β_i^{λ}). The relative separation distance R, in terms of the minimum surface separation distance s_0, is $R=s_0+R_1(\alpha_1)\cos\psi_1+R_2(\alpha_2)\cos\psi_2$. For a fixed R, s_0 is different for different orientations. Alternatively, for fixed s_0, R is different for different orientations, and we use this latter one in the following.

Fig. 1(b) illustrates for prolate-oblate ^{238}Pu+ ^{48}Ar reaction, the scattering potentials at different orientations (for λ=2,3,4 and λ=2 alone). An interesting result is that the barrier is lowered in each case except for 90^0-90^0 configuration, and that the barrier is lowest for 0^0-90^0 configuration. Note that for prolate-prolate collisions, the barrier is lowest for 0^0-180^0 configuration [14]. Thus, in the following, we neglect the 90^0-90^0 configuration since for fusion it is as un-favorable as the spherical nuclei. Also, note that the inclusion of higher multipole deformations is not always favored (barriers lowered) since for 45^0-135^0 and 45^0-90^0 orientations the barrier gets raised, rather than lowered as in 0^0-90^0, 0^0-180^0 and 90^0-90^0 (see Fig. 1(b) for 45^0-135^0 and 0^0-90^0 cases).

Fig. 2(a) illustrates the fragmentation potentials for various orientations of different target + projectile combinations at a fixed separation s_0=1.0 fm, forming the same compound nucleus $^{286}112^*$ (for 0^0-180^0 s_0=1.5 fm, since in this configuration the nuclei come much closer to each other). Apparently, due to the orientation degree of freedom, the energies of all the potential minima are lowered and the largest effect of the higher multipoles is for 0^0-180^0 which though is not the most favorable orientation for fusion (barrier not lowest) . We concentrate here only on the ^{48}Ca or ^{50}Ca minimum (marked by vertical lines; for some orientations, Ca changes to Ar). We notice that w.r.t. ground state energy (also marked), the Ca and/ or Ar minima have now the excitation energies E^*~15-20 MeV compared to ~ 30 MeV for spherical nuclei. This means that Ca or Ar beam could be used for cold fusion reactions, if the colliding target nuclei are oriented, a result obtained for the first time. Note that ^{50}Ca is also a radioactive nucleus and all orientations and higher multipole deformations are not always favorable for the fusion process.

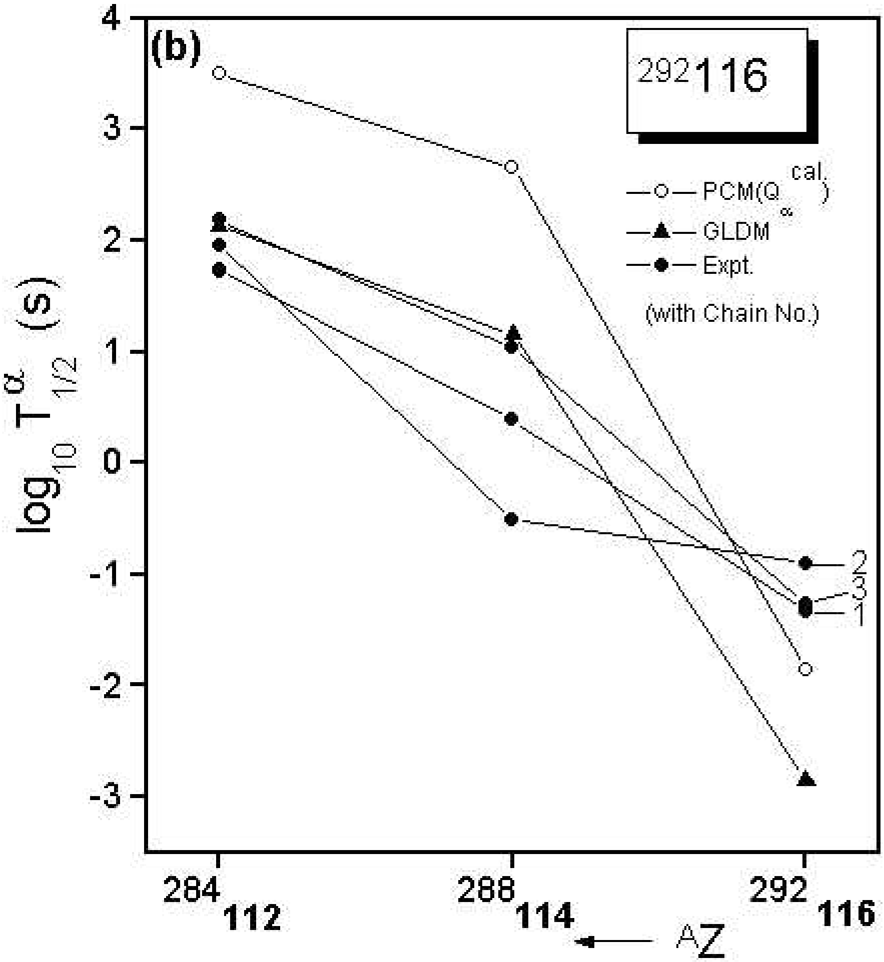

Figure 2. (a) Fragmentation potentials of $^{286}112^{*}$ for various orientations of different target + projectile combinations with λ=2,3,4 and λ=2 alone. For Z≤ 7, β_i^2 are from RMF with TM2 force [17]. (b) Calculated half-lives for α-decay chain of $^{292}116$, compared with experiments and GLDM calculation.

For α-decay studies, the preformed cluster-decay model (PCM) used here is also based on QMFT and hence on the same coordinates as are introduced above. In a PCM, the decay half-life is defined as,

$$T_{1/2} = \frac{\ln 2}{P_0 \nu_0 P} \tag{4}$$

The P_0, the cluster (and daughter) preformation probabilities in the ground state of nucleus referring to η-motion, are the solutions of the stationary Schrödinger equation (2) for the ground-state ν=0, and P, referring to R-motion, is WKB tunneling penetrability. The ν_0 is the barrier assault frequency. For details, see Refs. [9-11].

Fig. 2(b) illustrates our results of calculation for α-decay chain of $^{292}116$ parent [18], compared with the experimental data and another recent calculation [19], denoted GLDM. The numbers 1,2,3 in the figure mean that more than one chain is observed. We notice that the comparisons of $T_{1/2}$ values for the two models with experiments are within experimental errors, i.e. within less than two orders of magnitude. Both model calculations give similar trends.

Summarizing, we have extended the QMFT for use of oriented collisions and including higher multipole deformations, which result in the reduction of excitation energies of the compound system formed due to different target-projectile combinations. This means that both the "warm" and "hot" fusion reactions could also be reached in "cold fusion", if the target and/or projectile were oriented and have also octu- and hexa-decapole deformations. The QMFT based PCM is also shown to explain the α-decay characteristics of SHE.

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Four-dimensional dynamical approach to the multi-modal fission

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In the low energy fission of heavy nuclei, namely actinides and transactinides, it has been demonstrated that there exist more than one deformation paths for fission process. In mass distribution of fission fragments, mass-asymmetric fission is observed on top of the rather broad symmetric fission distribution. The mass-symmetric component decreases as the excitation energy of the compound nucleus increases, and it is expected that the shell correction is the origin of the asymmetric fission. Sometimes, the shape of the mass distribution shows a drastic change with the change of N (neutron number) and of Z (proton number). In the total kinetic energy (TKE) distribution, several components have been observed and it is expected that TKE reflects the compactness of the scission configuration.

The problem of fission modes has been studied theoretically as well [1, 2]. Because of the importance of the shell correction for the appearance of the multi-modal fission, the nuclear energy surface in a multi-dimensional deformation space has been investigated first to obtain static fission paths. Strutinsky◎fs shell correction method has been used to estimate the shell correction energy. Saddle points are found in the deformation space and the fission paths are defined as valley paths.

Recently, we proposed a dynamical approach to this problem. The multi-dimensional Langevin equation has been applied to the study of fission of highly excited nuclei and succeeded in reproducing the experimental data like pre-scission particle multiplicities and TKE distribution [3]. Importance of the nuclear friction has been stressed in these studies. By including the shell correction energy to the potential energy surface, we proposed to apply this method to the fission of low excitation as well.

By solving the Langevin equation in multi-dimensional deformation space, the compound nucleus finds its way to fission automatically without assuming the valley paths. Langevin trajectories go out of the spherical region through saddle points; each saddle point is selected according to its barrier height automatically. After passing through the saddles, the trajectories go down the nuclear potential energy surface and they reach the scission points. By looking at the shapes at scission points and by tracing the paths, we can easily distinguish the fission modes for each trajectory.

We applied this approach to several systems, like heavy actinides (Fm, Bk) and transactinides (Sg) [4]. We used a three-dimensional deformation space, namely we employed elongation, fragment deformation and mass-asymmetry to describe the nuclear shape. Since we restrict the model space to be three-dimensional, we put a constraint on the deformations of two future fragments; we assume that both fragments have the same deformation. The shell correction energy is calculated with the code TWOCTR [5]. We assume the hydrodynamical inertia mass and the one-body wall-and-window friction. In the study of Sg and Bk, we found a mass-asymmetric fission component together with a mass-symmetric one. We did not find a mass-asymmetric valley in the potential energy surface and concluded that this mass-asymmetric component appears as a result of the multi-dimensional dynamics. In the study of Fm, we found at least three modes: a compact mass-symmetric mode that corresponds to the magic daughter nucleus Sn, a mass-asymmetric component that has the same origin as in the case of Bk and Sg, and an elongated mass-symmetric component. The TKE value obtained in the theoretical study agreed with the experimental systematics well [6]. However, in these studies, we could not reproduce the experimental fractions of the mass-symmetric and mass-asymmetric components.

In the present paper, we extend the model space to four-dimensional one; we use independent deformation for each fragment. In this way, we can take account of the shell correction more precisely, especially for spherical shells that are strongest. We apply this four-dimensional approach for several heavy-actinides and transactinides.

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